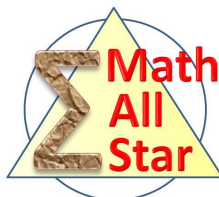


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# Trigonometry

## Basics Reinforcement

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*Math for Gifted Students*

<http://www.mathallstar.org>




# Basics Reinforcement



## Instructions

- Write down and submit intermediate steps along with your final answer.
- If the final result is too complex to compute, give the expression. e.g.  $C_{100}^{50}$  is acceptable.
- Problems are not necessarily ordered based on their difficulty levels.
- Always ask yourself what makes this problem a good practice?
- Read through the reference solution even if you can solve the problem for additional information which may help you to solve this type of problems.

## Legends

-  *Tips, additional information etc*
-  *Important theorem, conclusion to remember.*
-  *Addition questions for further study.*

## My Comments and Notes

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## Practice 1

Convert between radian and degree measures:

- |                  |                        |                        |
|------------------|------------------------|------------------------|
| (i) $30^\circ$   | (v) $120^\circ$        | (viii) $\frac{\pi}{6}$ |
| (ii) $45^\circ$  | (vi) $270^\circ$       | (ix) $-\frac{3\pi}{5}$ |
| (iii) $60^\circ$ | (vii) $-\frac{\pi}{4}$ | (x) $2\pi$             |
| (iv) $90^\circ$  |                        |                        |

## Practice 2

Complete the following table:

	$0^\circ$	$30^\circ$	$45^\circ$	$60^\circ$	$90^\circ$	$120^\circ$	$150^\circ$	$270^\circ$	$360^\circ$	$540^\circ$
sin										
cos										
tan										

## Practice 3

Which of the following equations always hold?

- |  |   |
|--|---|
| (i) $\sin^2 \theta + \cos^2 \theta = 1$        | (v) $\sin\left(\frac{\pi}{2} - \theta\right) = \cos \theta$   |
| (ii) $\tan \theta = \cos \theta / \sin \theta$ | (vi) $\sin\left(\frac{\pi}{2} + \theta\right) = -\cos \theta$ |
| (iii) $\sin(-\theta) = \sin \theta$            | (vii) $1 + \tan^2 \theta = \frac{1}{\cos^2 \theta}$           |
| (iv) $\cos(\pi - \theta) = \cos \theta$        |   |

## Practice 4

What are the ranges of the *sin*, *cos*, and *tan* function, respectively?

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**Practice 5**

When  $\theta$  increases from 0 to  $\frac{\pi}{2}$ , determine whether each statement below is true or not:

- (i) the value of  $\sin \theta$  increases
- (ii) the value of  $\cos \theta$  increases
- (iii) the value of  $\tan \theta$  increases

**Practice 6**

When  $\frac{\pi}{4} < \theta < \frac{\pi}{2}$ , which of the following statement holds?

- (i)  $\sin \theta > \cos \theta > \tan \theta$
- (ii)  $\cos \theta > \tan \theta > \sin \theta$
- (iii)  $\tan \theta > \sin \theta > \cos \theta$
- (iv)  $\sin \theta > \tan \theta > \cos \theta$

**Practice 7**

Let  $x$  be a real number and  $0 \leq x \leq \frac{\pi}{2}$ , explain why the following inequality holds and when the equality sign holds:

$$\sin x \leq x \leq \tan x$$

**Practice 8**

Find all angles  $\theta \in [0, 2\pi)$  such that  $\sin \theta = \frac{1}{2}$ . Express your answer in radian.

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**Practice 9**

Find all angles  $\theta$  such that  $\sin \theta = \frac{1}{2}$ . Express your answer in radian.

**Practice 10**

Find all angles  $\theta$  such that  $\sin \theta \leq \frac{1}{2}$ . Express your answer in radian.



## Answer Keys

Practice 1:

- |                                  |                                    |                                    |
|----------------------------------|------------------------------------|------------------------------------|
| (i) $30^\circ = \frac{\pi}{6}$   | (v) $120^\circ = \frac{2\pi}{3}$   | (viii) $\frac{\pi}{6} = 30^\circ$  |
| (ii) $45^\circ = \frac{\pi}{4}$  | (vi) $270^\circ = \frac{3\pi}{2}$  | (ix) $-\frac{3\pi}{5} = 108^\circ$ |
| (iii) $60^\circ = \frac{\pi}{3}$ | (vii) $-\frac{\pi}{4} = -45^\circ$ | (x) $2\pi = 360^\circ$             |
| (iv) $90^\circ = \frac{\pi}{2}$  |                                    |                                    |

Practice 2:

	$0^\circ$	$30^\circ$	$45^\circ$	$60^\circ$	$90^\circ$	$120^\circ$	$150^\circ$	$270^\circ$	$360^\circ$	$540^\circ$
sin	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	-1	0	0
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	0	1	-1
tan	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	$+\infty$	$-\sqrt{3}$	$-\frac{\sqrt{3}}{3}$	$-\infty$	0	0

Practice 3:

- |             |            |
|-------------|------------|
| (i) TRUE    | (v) TRUE   |
| (ii) FALSE  | (vi) TRUE  |
| (iii) FALSE | (vii) TRUE |
| (iv) FALSE  |            |

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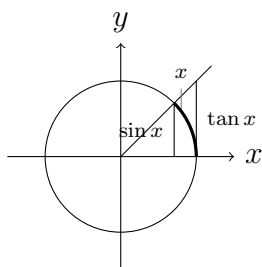
Practice 4:  $[-1, 1]$ ,  $[-1, 1]$ ,  $(-\infty, +\infty)$

Practice 5:

- (i) True
- (ii) False
- (iii) True

Practice 6: (iii)

Practice 7:



The equality holds when  $x = 0$ .

Practice 8:  $\theta = \frac{\pi}{6}, \frac{5\pi}{6}$

Practice 9:  $\theta = 2k\pi + \frac{\pi}{6}, 2k\pi + \frac{5\pi}{6}$  where  $k$  is an integer, or

$\theta = k\pi + (-1)^k \cdot \frac{\pi}{6}$  where  $k$  is an integer.

Practice 10:

$\theta \in [2k\pi, (2k + \frac{1}{6})\pi] \cup [2k\pi + \frac{5\pi}{6}, (2k + 2)\pi]$ , where  $k \in \mathbb{Z}$ , or

$\theta \in [(2k - \frac{7}{6})\pi, (2k + \frac{1}{6})\pi]$ , where  $k \in \mathbb{Z}$

