

Counting

## Symmetry and Bundling



Learn how to solve this *type* of problems, not just this problem.

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*Tip: Always write down intermediate steps.*

- (1) Can you explain why  $C_n^k = C_n^{n-k}$  holds using symmetry argument?
- (2) Six people form a line.  $A$  must stand after  $B$  (not necessarily immediately after  $B$ ). How many different ways are there to form such a line?  
(Ref: 2523)
- (3) Ten people form a line, among which two are Chinese and two are Americans. The probability that both Chinese will stand in front of both Americans (not necessarily immediately in the front) can be expressed as  $\frac{p}{q}$  where  $p$  and  $q$  are two relatively prime positive integers. Find  $p + q$   
(Ref: 3)
- (4) Seven people form a line. If  $A$  must stand next to  $B$ , and  $C$  must stand next to  $D$ , how many possibilities are there?  
(Ref: 2524)
- (5) A spider has one sock and one shoe for each of its eight legs. In how many different orders can the spider put on its socks and shoes, assuming that, on each leg, the sock must be put on before the shoe.  
(Ref: 2191 - 2001 AMC12 #16)
- (6) Fifteen guards and five prisoners stand in a single row. To ensure security, every prisoner must be escorted by two guards, one on each side. The five extra guards can stand anywhere in the row. How many different arrangements can be made?
- (7) Joe and Mary flip a coin  $(n + 1)$  and  $n$  times, respectively. What is the probability that Joe gets more heads than Mary does?
- (8) Ten unfair coins with probability of  $1, \frac{1}{2}, \frac{1}{3}, \dots, \frac{1}{10}$  of showing heads are flipped. What is the probability that odd number of heads are shown?  
(2248)