Pell's Equation

Learn how to solve this *type* of problems, not just this problem.

1. Solve in integers the equation $x^2 + y^2 - 1 = 4xy$.
   (Ref Ref 2300)

2. Show that if x and y are positive integer solutions to the equation $x^2 - 2y^2 = 1$, then $6 \mid xy$.
   (Ref 2093)

3. Find all right triangles whose two sides are consecutive integers, and hypotenuse are also integers.
   (Ref Ref 2347)

4. Find all triangles whose sides are consecutive integers and areas are also integers.
   (Ref Ref 2096)

5. Find all positive integers k, m such that $k < m$ and
   $$1 + 2 + \cdots + k = (k+1) + (k+2) + \cdots + m$$
   (Ref Ref 2098)

6. Find all positive integer n such that
   $$C_n^{k-1} = 2C_n^k + C_n^{k+1}$$
   for some positive integer $k < n$.
   (Ref Ref 2306)

7. Prove that if the difference of two consecutive cubes is n^2 where n is a positive integer, then $2n - 1$ is a square.
   (Ref Ref 2308)

8. Let r be a positive real number, and $[r]$ be the largest integer that not exceeding x. Prove that there exist infinity number of positive integers, n, such that $[\sqrt{2}\, n]$ is a perfect square.
   (Ref Ref 2302)

9. Solve in *rational* numbers the equation $x^2 - dy^2 = 1$ where d is an integer.
   (Ref Ref 2301)

10. Let p be a prime. Prove that the equation $x^2 - py^2 = -1$ has integral solution if and only if $p = 2$ or $p \equiv 1 \pmod 4$.
    (Ref Ref 2310)