Counting

Recursion

Learn how to solve this *type* of problems, not just this problem.

💡 *Tip: Always write down intermediate steps.*

(1) Joe wants to write 1 to n in a $1 \times n$ grid. The number 1 can be written in any grid, while the number 2 must be written next to 1 (can be at either side) so that these two numbers are together. The number 3 must be written next to this two-number block. This process goes on. Every new number written must stay next to the existing number block. How many different ways can Joe fill this $1 \times n$ grid?

(Ref: 2479)

(2) Cozy the Cat is going up a staircase of 10 steps. She can either walk up 1 step a time or jump 2 steps a time. How many different ways can she reach the top of this staircase?

(2069)

(3) In a small pond there are eleven lily pads in a row labeled 0 through 10. A frog is sitting on pad 1. When the frog is on pad N, $0 < N < 10$, it will jump to pad $N - 1$ with probability $\frac{N}{10}$ and to pad $N + 1$ with probability $1 - \frac{N}{10}$. Each jump is independent of the previous jumps. If the frog reaches pad 0 it will be eaten by a patiently waiting snake. If the frog reaches pad 10 it will exit the pond, never to return. What is the probability that the frog will escape being eaten by the snake?

(A) $\frac{32}{79}$ **(B)** $\frac{161}{384}$ **(C)** $\frac{63}{146}$ **(D)** $\frac{7}{16}$ **(E)** $\frac{1}{2}$

(Ref: 1329 - 2014 AMC10B #25, AMC12B #22)

(4) Joe divides a circle into n sectors and colors each of these sectors using one of k different colors ($k \geq 3$). How many coloring schemes are there such that no neighboring sectors have the same color?

(Ref 2737)

(5) There are n circles on the plane. Every pair of two circles intersect at two points. No three circles pass the same point. How many regions do these circles divide the whole plane into?

(Ref 2738)