

The Inequality (Squeeze) Method



Learn how to solve this *type* of problems, not just this problem.

1. Solve this equation in integers: $y^2 = x^2 + x + 1$.
2. Solve in integers the equation $y^2 = x^4 + x^3 + x^2 + x + 1$.
(Ref Ref 2081)

3. Solve in integers the equation

$$\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = \frac{3}{5}$$

(Ref Ref 2084: Romania Olympiad)

4. Solve in positive integers the equation

$$3(xy + yz + zx) = 4xyz$$

(Ref Ref 2243: Putname)

5. A rectangular box measures $a \times b \times c$, where a , b , and c are integers and $1 \leq a \leq b \leq c$. The volume and the surface area of the box are numerically equal. How many ordered triples (a, b, c) are possible?

(Ref Ref 401: 2015 AMC12B #23)

6. Solve in integers the equation

$$(x + y)^2 = x^3 + y^3$$

(Ref Ref 2359)

7. How many ordered triples of integers (a, b, c) , with $a \geq 2$, $b \geq 1$, and $c \geq 0$, satisfy both $\log_a b = c^{2005}$ and $a + b + c = 2005$?

(Ref Ref 914: 2005 AMC12A #21)

8. Find all integers a , b , c with $1 < a < b < c$ such that the number $(a - 1)(b - 1)(c - 1)$ is a divisor of $abc - 1$.

(Ref Ref 2240: 1992 IMO)

9. Find all positive integers n and k_i ($1 \leq i \leq n$) such that

$$k_1 + k_2 + \cdots + k_n = 5n - 4$$

and

$$\frac{1}{k_1} + \frac{1}{k_2} + \cdots + \frac{1}{k_n} = 1$$

(Ref Ref 2242: Putnam)

10. Solve in positive integers $(1 + \frac{1}{x})(1 + \frac{1}{y})(1 + \frac{1}{z}) = 2$

(Ref Ref 2241: UK Olympiad)