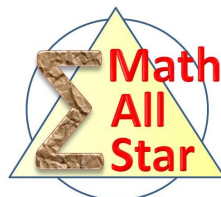

Trigonometry

Basics Reinforcement



Math for Gifted Students

<http://www.mathallstar.org>

Basics Reinforcement



Instructions

- Write down and submit intermediate steps along with your final answer.
- If the final result is too complex to compute, give the expression. e.g. C_{100}^{50} is acceptable.
- Problems are not necessarily ordered based on their difficulty levels.
- Always ask yourself what makes this problem a good practice?
- Read through the reference solution even if you can solve the problem for additional information which may help you to solve this type of problems.

Legends



Tips, additional information etc



Important theorem, conclusion to remember.



Addition questions for further study.

My Comments and Notes

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Practice 1

Convert between radian and degree measures:

- | | | |
|------------------|------------------------|------------------------|
| (i) 30° | (v) 120° | (viii) $\frac{\pi}{6}$ |
| (ii) 45° | (vi) 270° | (ix) $-\frac{3\pi}{5}$ |
| (iii) 60° | (vii) $-\frac{\pi}{4}$ | (x) 2π |
| (iv) 90° | | |

Practice 2

Complete the following table:

| | 0° | 30° | 45° | 60° | 90° | 120° | 150° | 270° | 360° | 540° |
|-----|-----------|------------|------------|------------|------------|-------------|-------------|-------------|-------------|-------------|
| sin | | | | | | | | | | |
| cos | | | | | | | | | | |
| tan | | | | | | | | | | |

Practice 3

Which of the following equations always hold?

- | | |
|--|---|
| (i) $\sin^2 \theta + \cos^2 \theta = 1$ | (v) $\sin\left(\frac{\pi}{2} - \theta\right) = \cos \theta$ |
| (ii) $\tan \theta = \cos \theta / \sin \theta$ | (vi) $\sin\left(\frac{\pi}{2} + \theta\right) = -\cos \theta$ |
| (iii) $\sin(-\theta) = \sin \theta$ | (vii) $1 + \tan^2 \theta = \frac{1}{\cos^2 \theta}$ |
| (iv) $\cos(\pi - \theta) = \cos \theta$ | |

Practice 4

What are the ranges of the \sin , \cos , and \tan function, respectively?

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Practice 5

When θ increases from 0 to $\frac{\pi}{2}$, determine whether each statement below is true or not:

- (i) the value of $\sin \theta$ increases
- (ii) the value of $\cos \theta$ increases
- (iii) the value of $\tan \theta$ increases

Practice 6

When $\frac{\pi}{4} < \theta < \frac{\pi}{2}$, which of the following statement holds?

- (i) $\sin \theta > \cos \theta > \tan \theta$
- (ii) $\cos \theta > \tan \theta > \sin \theta$
- (iii) $\tan \theta > \sin \theta > \cos \theta$
- (iv) $\sin \theta > \tan \theta > \cos \theta$

Practice 7

Let x be a real number and $0 \leq x \leq \frac{\pi}{2}$, explain why the following inequality holds and when the equality sign holds:

$$\sin x \leq x \leq \tan x$$

Practice 8

Find all angles $\theta \in [0, 2\pi)$ such that $\sin \theta = \frac{1}{2}$. Express your answer in radian.

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Practice 9

Find all angles θ such that $\sin \theta = \frac{1}{2}$. Express your answer in radian.

Practice 10

Find all angles θ such that $\sin \theta \leq \frac{1}{2}$. Express your answer in radian.

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Answer Keys

Practice 1:

(i) $30^\circ = \frac{\pi}{6}$

(v) $120^\circ = \frac{2\pi}{3}$

(viii) $\frac{\pi}{6} = 30^\circ$

(ii) $45^\circ = \frac{\pi}{4}$

(vi) $270^\circ = \frac{3\pi}{2}$

(ix) $-\frac{3\pi}{5} = 108^\circ$

(iii) $60^\circ = \frac{\pi}{3}$

(vii) $-\frac{\pi}{4} = -45^\circ$

(x) $2\pi = 360^\circ$

(iv) $90^\circ = \frac{\pi}{2}$

Practice 2:

| | 0° | 30° | 45° | 60° | 90° | 120° | 150° | 270° | 360° | 540° |
|-----|-----------|----------------------|----------------------|----------------------|------------|----------------------|-----------------------|-------------|-------------|-------------|
| sin | 0 | $\frac{1}{2}$ | $\frac{\sqrt{2}}{2}$ | $\frac{\sqrt{3}}{2}$ | 1 | $\frac{\sqrt{3}}{2}$ | $\frac{1}{2}$ | -1 | 0 | 0 |
| sin | 1 | $\frac{\sqrt{3}}{2}$ | $\frac{\sqrt{2}}{2}$ | $\frac{1}{2}$ | 0 | $-\frac{1}{2}$ | $-\frac{\sqrt{3}}{2}$ | 0 | 1 | -1 |
| tan | 0 | $\frac{\sqrt{3}}{3}$ | 1 | $\sqrt{3}$ | $+\infty$ | $-\sqrt{3}$ | $-\frac{\sqrt{3}}{3}$ | $-\infty$ | 0 | 0 |

Practice 3:

(i) TRUE

(v) TRUE

(ii) FALSE

(vi) TRUE

(iii) FALSE

(vii) TRUE

(iv) FALSE

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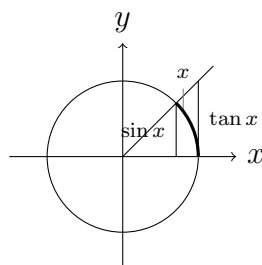
Practice 4: $[-1, 1]$, $[-1, 1]$, $(-\infty, +\infty)$

Practice 5:

- (i) True
- (ii) False
- (iii) True

Practice 6: (iii)

Practice 7:



The equality holds when $x = 0$.

Practice 8: $\theta = \frac{\pi}{6}, \frac{5\pi}{6}$

Practice 9: $\theta = 2k\pi + \frac{\pi}{6}, 2k\pi + \frac{5\pi}{6}$ where k is an integer, or

$\theta = k\pi + (-1)^k \cdot \frac{\pi}{6}$ where k is an integer.

Practice 10:

$\theta \in [2k\pi, (2k + \frac{1}{6})\pi] \cup [2k\pi + \frac{5\pi}{6}, (2k + 2)\pi]$, where $k \in \mathbb{Z}$, or
 $\theta \in [(2k - \frac{7}{6})\pi, (2k + \frac{1}{6})\pi]$, where $k \in \mathbb{Z}$

