Basics Reinforcement



Math for Gifted Students

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Instructions

- Write down and submit intermediate steps along with your final answer.
- If the final result is too complex to compute, give the expression. e.g. C_{100}^{50} is acceptable.
- Problems are not necessarily ordered based on their difficulty levels.
- Always ask yourself what makes this problem a good practice?
- Read through the reference solution even if you can solve the problem for additional information which may help you to solve this type of problems.

Legends

- Tips, additional information etc
- ✓ Important theorem, conclusion to remember.
- Addition questions for further study.

My Comments and Notes

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Practice 1

Convert between radian and degree measures:

(i)
$$30^{\circ}$$

(v)
$$120^{\circ}$$

(viii)
$$\frac{\pi}{6}$$

(ix)
$$-\frac{3\pi}{5}$$

(iii)
$$60^{\circ}$$

(vii)
$$-\frac{\pi}{4}$$

$$(x)$$
 2π

Practice 2

(iv)

Complete the following table:

 90°

	0°	30°	45°	60°	90°	120°	150°	270°	360°	540°
sin										
cos										
tan										

Practice 3

Which of the following equations always hold?

(i)
$$\sin^2\theta + \cos^2\theta = 1$$

$$(v) \quad \sin(\frac{\pi}{2} - \theta) = \cos\theta$$

(ii)
$$\tan \theta = \cos \theta / \sin \theta$$

(vi)
$$\sin(\frac{\pi}{2} + \theta) = -\cos\theta$$

(iii)
$$\sin(-\theta) = \sin \theta$$

(vii)
$$1 + \tan^2 \theta = \frac{1}{\cos^2 \theta}$$

(iv) $\cos(\pi - \theta) = \cos \theta$

Practice 4

What are the ranges of the sin, cos, and tan function, respectively?

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Practice 5

When θ increases from 0 to $\frac{\pi}{2}$, determine whether each statement below is true or not:

- (i) the value of $\sin \theta$ increases
- (ii) the value of $\cos \theta$ increases
- (iii) the value of $\tan \theta$ increases

Practice 6

When $\frac{\pi}{4} < \theta < \frac{\pi}{2}$, which of the following statement holds?

- (i) $\sin \theta > \cos \theta > \tan \theta$
- (ii) $\cos \theta > \tan \theta > \sin \theta$
- (iii) $\tan \theta > \sin \theta > \cos \theta$
- (iv) $\sin \theta > \tan \theta > \cos \theta$

Practice 7

Let x be a real number and $0 \le x \le \frac{\pi}{2}$, explain why the following inequality holds and when the equality sign holds:

$$\sin x \le x \le \tan x$$

Practice 8

Find all angles $\theta \in [0, 2\pi)$ such that $\sin \theta = \frac{1}{2}$. Express your answer in radian.

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Practice 9

Find all angles θ such that $\sin \theta = \frac{1}{2}$. Express your answer in radian.

Practice 10

Find all angles θ such that $\sin \theta \leq \frac{1}{2}$. Express your answer in radian.

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Answer Keys

Practice 1:

(i)
$$30^{\circ} = \frac{\pi}{6}$$

(v)
$$120^{\circ} = \frac{2\pi}{3}$$

(viii)
$$\frac{\pi}{6} = 30^{\circ}$$

(ii)
$$45^{\circ} = \frac{\pi}{4}$$

(iii)
$$60^{\circ} = \frac{\pi}{3}$$

(vi)
$$270^{\circ} = \frac{3\pi}{2}$$

$$(ix) \qquad -\frac{3\pi}{5} = 108^{\circ}$$

(iv)
$$90^{\circ} = \frac{\pi}{2}$$

(vii)
$$-\frac{\pi}{4} = -45^{\circ}$$

(x)
$$2\pi = 360^{\circ}$$

Practice 2:

	0°	30°	45°	60°	90°	120°	150°	270°	360°	540°
sin	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	-1	0	0
sin	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	0	1	-1
tan	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	$+\infty$	$-\sqrt{3}$	$-\frac{\sqrt{3}}{3}$	$-\infty$	0	0

Practice 3:

(i) TRUE (v) TRUE

(ii) **FALSE**

(vi) TRUE

FALSE (iii)

(iv) FALSE

TRUE (vii)

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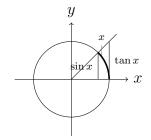
Practice 4: $[-1, 1], [-1, 1], (-\infty, +\infty)$

Practice 5:

- (i) True
- (ii) False
- (iii) True

Practice 6: (iii)

Practice 7:



The equality holds when x = 0.

Practice 8: $\theta = \frac{\pi}{6}, \frac{5\pi}{6}$

Practice 9: $\theta = 2k\pi + \frac{\pi}{6}$, $2k\pi + \frac{5\pi}{6}$ where k is an integer, or $\theta = k\pi + (-1)^k \cdot \frac{\pi}{6}$ where k is an integer.

Practice 10:

 $\theta \in [2k\pi, (2k + \frac{1}{6})\pi] \cup [2k\pi + \frac{5\pi}{6}, (2k + 2)\pi], \text{ where } k \in \mathbb{Z}, \text{ or } \theta \in [(2k - \frac{7}{6})\pi, (2k + \frac{1}{6})\pi], \text{ where } k \in \mathbb{Z}$

