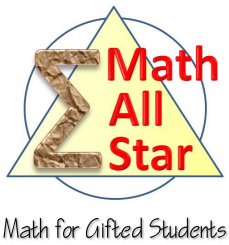
Trigonometry

Basics Reinforcement



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Instructions

- *Write down and submit intermediate steps along with your final answer.*
- *If the final result is too complex to compute, give the expression. e.g.* C_{100}^{50} *is acceptable.*
- *Problems are not necessarily ordered based on their difficulty levels.*
- *Always ask yourself what makes this problem a good practice?*
- *Read through the reference solution even if you can solve the problem for additional information which may help you to solve this type of problems.*

Legends

Tips, additional information etc

Important theorem, conclusion to remember.

Addition questions for further study.

My Comments and Notes

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Practice 1

Convert between radian and degree measures:

(i) $30°$

(ii) $45°$

(iii) $60°$

(iv) $90°$

(v) $120°$

(vi) $270°$

(vii) $-\frac{\pi}{4}$

(viii) $\frac{\pi}{6}$

(ix) $-\frac{3\pi}{5}$

(x) 2π

Practice 2

Complete the following table:

	$0°$	$30°$	$45°$	$60°$	$90°$	$120°$	$150°$	$270°$	$360°$	$540°$
sin										
cos										
tan										

Practice 3

Which of the following equations always hold?

(i) $\sin^2\theta + \cos^2\theta = 1$

(ii) $\tan\theta = \cos\theta / \sin\theta$

(iii) $\sin(-\theta) = \sin\theta$

(iv) $\cos(\pi - \theta) = \cos\theta$

(v) $\sin(\frac{\pi}{2} - \theta) = \cos\theta$

(vi) $\sin(\frac{\pi}{2} + \theta) = -\cos\theta$

(vii) $1 + \tan^2\theta = \frac{1}{\cos^2\theta}$

Practice 4

What are the ranges of the *sin*, *cos*, and *tan* function, respectively?

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Practice 5

When θ increases from 0 to $\dfrac{\pi}{2}$, determine whether each statement below is true or not:

(i) the value of $\sin\theta$ increases

(ii) the value of $\cos\theta$ increases

(iii) the value of $\tan\theta$ increases

Practice 6

When $\dfrac{\pi}{4} < \theta < \dfrac{\pi}{2}$, which of the following statement holds?

(i) $\sin\theta > \cos\theta > \tan\theta$

(ii) $\cos\theta > \tan\theta > \sin\theta$

(iii) $\tan\theta > \sin\theta > \cos\theta$

(iv) $\sin\theta > \tan\theta > \cos\theta$

Practice 7

Let x be a real number and $0 \leq x \leq \dfrac{\pi}{2}$, explain why the following inequality holds and when the equality sign holds:

$$\sin x \leq x \leq \tan x$$

Practice 8

Find all angles $\theta \in [0, 2\pi)$ such that $\sin\theta = \dfrac{1}{2}$. Express your answer in radian.

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Practice 9

Find all angles θ such that $\sin\theta = \frac{1}{2}$. Express your answer in radian.

Practice 10

Find all angles θ such that $\sin\theta \leq \frac{1}{2}$. Express your answer in radian.

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Answer Keys

Practice 1:

(i) $30° = \frac{\pi}{6}$

(ii) $45° = \frac{\pi}{4}$

(iii) $60° = \frac{\pi}{3}$

(iv) $90° = \frac{\pi}{2}$

(v) $120° = \frac{2\pi}{3}$

(vi) $270° = \frac{3\pi}{2}$

(vii) $-\frac{\pi}{4} = -45°$

(viii) $\frac{\pi}{6} = 30°$

(ix) $-\frac{3\pi}{5} = 108°$

(x) $2\pi = 360°$

Practice 2:

	$0°$	$30°$	$45°$	$60°$	$90°$	$120°$	$150°$	$270°$	$360°$	$540°$
sin	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	-1	0	0
sin	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	0	1	-1
tan	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	$+\infty$	$-\sqrt{3}$	$-\frac{\sqrt{3}}{3}$	$-\infty$	0	0

Practice 3:

(i) TRUE

(ii) FALSE

(iii) FALSE

(iv) FALSE

(v) TRUE

(vi) TRUE

(vii) TRUE

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Practice 4: $[-1, 1]$, $[-1, 1]$, $(-\infty, +\infty)$

Practice 5:

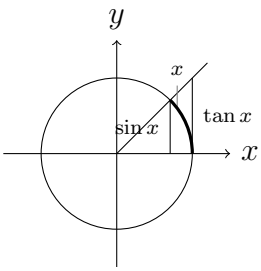
(i) True

(ii) False

(iii) True

Practice 6: (iii)

Practice 7:



The equality holds when $x = 0$.

Practice 8: $\theta = \dfrac{\pi}{6}$, $\dfrac{5\pi}{6}$

Practice 9: $\theta = 2k\pi + \dfrac{\pi}{6}$, $2k\pi + \dfrac{5\pi}{6}$ where k is an integer, or

$\theta = k\pi + (-1)^k \cdot \dfrac{\pi}{6}$ where k is an integer.

Practice 10:

$\theta \in [2k\pi, (2k + \dfrac{1}{6})\pi] \cup [2k\pi + \dfrac{5\pi}{6}, (2k + 2)\pi]$, where $k \in \mathbb{Z}$, or

$\theta \in [(2k - \dfrac{7}{6})\pi, (2k + \dfrac{1}{6})\pi]$, where $k \in \mathbb{Z}$