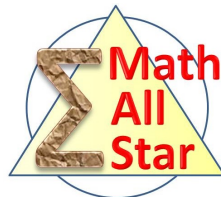

Art of Thinking

Pigeonhole Principle



Math for Gifted Students

<http://www.mathallstar.org>

Art of Thinking

Pigeonhole Principle



Instructions

- Write down and submit intermediate steps along with your final answer.
- If the final result is too complex to compute, give the expression. e.g. C_{100}^{50} is acceptable.
- Problems are not necessarily ordered based on their difficulty levels.
- Always ask yourself what makes this problem a good one to practise?
- Complete the My Record section below before submission.

My Comments and Notes

Sample

Pigeonhole Principle



Practice 1

With 13 randomly selected points inside a square of side length 2, show that there must exist quadrilateral whose vertices are among these 13 points and area is no more than 1.

(Ref 2793)

Practice 2

Show that among any four randomly selected integers, at least two of them must have a difference which is a multiple of 3.

Practice 3

Let $a_1, a_2, a_3, \dots, a_9$ be a random permutation of 1, 2, 3, ..., 9. Prove

$$(a_1 - 1)(a_2 - 2) \cdots (a_9 - 9)$$

is an even number.

(Ref UK Olympiad)

Practice 4

Prove: randomly select 51 numbers from 1, 2, 3, ..., 100, there must exist two numbers for which one is a multiple of the other.

(Ref 1119)

Practice 5

Show that it is possible to find an integer whose digits are all 8 and it is a multiple of 2016.

(Ref 2792)

Practice 6

Prove: any convex pentagon must have three vertices A , B , and C , such that $\angle ABC \leq 36^\circ$.

(Ref 2105)

Pigeonhole Principle



Battle Field

Here are some related problems selected from recent competitions:

Problem 1: 2012 MathCounts State Target #5

Problem 2: 2002 AMC10P #15

Sample