
Geometry

Coordinate Basics



Math for Gifted Students

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Coordinate Basics



Instructions

- Write down and submit intermediate steps along with your final answer.
- If the final result is too complex to compute, give the expression. e.g. C_{100}^{50} is acceptable.
- Problems are not necessarily ordered based on their difficulty levels.
- Always ask yourself what makes this problem a good practice?
- Read through the reference solution even if you can solve the problem for additional information which may help you to solve this type of problems.

Legends

-  *Tips, additional information etc*
-  *Important theorem, conclusion to remember.*
-  *Addition questions for further study.*

My Comments and Notes

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The emphasis of this practice is to write the required equations or formulas **directly**.



You should remember all the conclusions in this practice.

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Practice 1

(Distance) What is the distance between (x_1, y_1) and (x_2, y_2) ?

Practice 2

(Middle Point) What is the coordinate of the middle point between (x_1, y_1) and (x_2, y_2) ?

Practice 3

(Centroid) Give a triangle whose vertices are (x_1, y_1) , (x_2, y_2) , and (x_3, y_3) , respectively. What is the coordinate of its center of mass (centroid)?

Practice 4

(Incenter) Give a triangle whose vertices are $A(x_a, y_a)$, $B(x_b, y_b)$, and $C(x_c, y_c)$, respectively. If $a = BC$, $b = CA$, and $c = BA$, what is the coordinate of its incenter?

Practice 5

(Interpolation) Give two points $A(x_a, y_a)$ and $B(x_b, y_b)$, a point C on \overline{AB} . If $AC : CB = m : n$ where m and n are two integers, find the coordinate of point C .

Practice 6

(Line by Two Points) If a straight line l passes two distinct points (x_1, y_1) and (x_2, y_2) , what is l 's equation?

Practice 7

(Line by Intercepts) What is the equation of a straight line if its x -intercept and y -intercept are a and b , respectively?

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**Practice 8**

(Plane by Intercepts) What is the equation of a plane if its x -intercept, y -intercept, and z -intercept are a , b , and c , respectively?

Practice 9

(Point to Line) What is the distance from the point (x_0, y_0) to the line $Ax + By + C = 0$?

Practice 10

(Point to Plane) What is the distance from the point (x_0, y_0, z_0) to the plane $Ax + By + Cz + D = 0$?

Practice 11

(Triangle Area) What is the area of a triangle whose vertices are (x_1, y_1) , (x_2, y_2) , and (x_3, y_3) ?

Practice 12

(Polygon Area) What is the area of a polygon whose vertices are (x_1, y_1) , (x_2, y_2) , \dots and (x_n, y_n) ?

Practice 13

Given two parallel lines: $Ax + By + C_1 = 0$ and $Ax + By + C_2 = 0$, find the locus of all the points that are equidistant to these two lines.

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Practice 14

(Circle Tangent Line) Let point $P(x_0, y_0)$ be on circle O , find the equation of the straight line which is tangent to O on point P when O is given by:

(i) $x^2 + y^2 = r^2$

(ii) $(x - a)^2 + (y - b)^2 = r^2$

(iii) $x^2 + y^2 + Dx + Ey + F = 0$

Practice 15

(Chord Passing Tangent Points) Let point $P(x_0, y_0)$ be outside the circle $O : x^2 + y^2 = 0$. If PA and PB are two lines that pass P and are tangent to O at A and B , find the equation of line AB .

Practice 16

(Distance to Tangent Points) Let point $P(x_0, y_0)$ be outside the circle O . If PA is a line that passes P and is tangent to O at A , find the distance between P and A when O is given by:

(i) $x^2 + y^2 = r^2$

(ii) $(x - a)^2 + (y - b)^2 = r^2$

(iii) $x^2 + y^2 + Dx + Ey + F = 0$

Reference Solutions

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Practice 1

(Distance) What is the distance between (x_1, y_1) and (x_2, y_2) ?

$$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Practice 2

(Middle Point) What is the coordinate of the middle point between (x_1, y_1) and (x_2, y_2) ?

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

Practice 3

(Centroid) Give a triangle whose vertices are (x_1, y_1) , (x_2, y_2) , and (x_3, y_3) , respectively. What is the coordinate of its center of mass (centroid)?

$$\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3} \right)$$

Practice 4

(Incenter) Give a triangle whose vertices are $A(x_a, y_a)$, $B(x_b, y_b)$, and $C(x_c, y_c)$, respectively. If $a = BC$, $b = CA$, and $c = BA$, what is the coordinate of its incenter?

$$\left(\frac{ax_a + bx_b + cx_c}{a + b + c}, \frac{ay_a + by_b + cy_c}{a + b + c} \right)$$

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Practice 5

(Interpolation) Give two points $A(x_a, y_a)$ and $B(x_b, y_b)$, a point C on \overline{AB} . If $AC : CB = m : n$ where m and n are two integers, find the coordinate of point C .

$$\left(\frac{nx_a + mx_b}{m + n}, \frac{ny_a + my_b}{m + n} \right)$$

Practice 6

(Line by Two Points) If a straight line l passes two distinct points (x_1, y_1) and (x_2, y_2) , what is l 's equation?

$$\frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1} \quad \text{or} \quad \frac{y - y_2}{x - x_2} = \frac{y_2 - y_1}{x_2 - x_1}$$

Practice 7

(Line by Intercepts) What is the equation of a straight line if its x -intercept and y -intercept are a and b , respectively?

$$\frac{x}{a} + \frac{y}{b} = 1$$

Practice 8

(Plane by Intercepts) What is the equation of a plane if its x -intercept, y -intercept, and z -intercept are a , b , and c , respectively?

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$$

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Practice 9

(Point to Line) What is the distance from the point (x_0, y_0) to the line $Ax + By + C = 0$?

$$\frac{|Ax_0 + By_0 + C|}{\sqrt{A^2 + B^2}}$$

Practice 10

(Point to Plane) What is the distance from the point (x_0, y_0, z_0) to the plane $Ax + By + Cz + D = 0$?

$$\frac{|Ax_0 + By_0 + Cz_0 + D|}{\sqrt{A^2 + B^2 + C^2}}$$

Practice 11

(Triangle Area) What is the area of a triangle whose vertices are (x_1, y_1) , (x_2, y_2) , and (x_3, y_3) ?

$$\frac{|(x_1y_2 - y_1x_2) + (x_2y_3 - y_2x_3) + (x_3y_1 - y_3x_1)|}{2}$$

Practice 12

(Polygon Area) What is the area of a polygon whose vertices are (x_1, y_1) , (x_2, y_2) , \dots and (x_n, y_n) ?

$$\frac{|(x_1y_2 - y_1x_2) + (x_2y_3 - y_2x_3) + \dots + (x_ny_1 - y_nx_1)|}{2}$$

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Practice 13

Given two parallel lines: $Ax + By + C_1 = 0$ and $Ax + By + C_2 = 0$, find the locus of all the points that are equidistant to these two lines.

$$Ax + By + \frac{C_1 + C_2}{2} = 0$$

Practice 14

(Circle Tangent Line) Let point $P(x_0, y_0)$ be on circle O , find the equation of the straight line which is tangent to O on point P when O is given by:

(i) $x^2 + y^2 = r^2$

(ii) $(x - a)^2 + (y - b)^2 = r^2$

(iii) $x^2 + y^2 + Dx + Ey + F = 0$

(i) $x_0x + y_0y = r^2$

(ii) $(x_0 - a)(x - a) + (y_0 - b)(y - b) = r^2$

(iii) $x_0x + y_0y + D \cdot \frac{x_0 + x}{2} + E \cdot \frac{y_0 + y}{2} + F = 0$

Practice 15

(Chord Passing Tangent Points) Let point $P(x_0, y_0)$ be outside the circle $O : x^2 + y^2 = r^2$. If PA and PB are two lines that pass P and are tangent to O at A and B , find the equation of line AB .

$$x_0x + y_0y = r^2$$

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Practice 16

(Distance to Tangent Points) Let point $P(x_0, y_0)$ be outside the circle O . If PA is a line that passes P and is tangent to O at A , find the distance between P and A when O is given by:

(i) $x^2 + y^2 = r^2$

(ii) $(x - a)^2 + (y - b)^2 = r^2$

(iii) $x^2 + y^2 + Dx + Ey + F = 0$

(i) $\sqrt{x_0^2 + y_0^2 - r^2}$

(ii) $\sqrt{(x_0 - a)^2 + (y_0 - b)^2 - r^2}$

(iii) $\sqrt{x_0^2 + y_0^2 + Dx_0 + Ey_0 + F}$



Tip: How can you remember these three formulas?



Battle Field

Selective problems from recent competitions:

- Problem 1: 2016 AMC10B #20 (Ref 2925)
- Problem 2: 2015 AIME II #9 (Ref 76)
- Problem 3: 2015 AIME I #4 (Ref 56)
- Problem 4: 2013 AMC12A #13 (Ref 489)
- Problem 5: 2013 MathCounts State Target #6 (Ref 1841)
- Problem 6: 2012 AMC10B #23 (Ref 1427)
- Problem 7: 2012 MathCounts Chapter Target #3 (Ref 1975)
- Problem 8: 2012 MathCounts Chapter Team #2 (Ref 1982)