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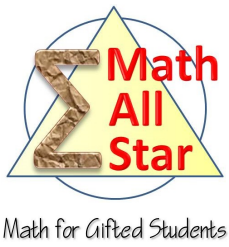
Geometry

Coordinate Basics



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Geometry

Coordinate Basics

Instructions

- *Write down and submit intermediate steps along with your final answer.*
- *If the final result is too complex to compute, give the expression. e.g.* C_{100}^{50} *is acceptable.*
- *Problems are not necessarily ordered based on their difficulty levels.*
- *Always ask yourself what makes this problem a good practice?*
- *Read through the reference solution even if you can solve the problem for additional information which may help you to solve this type of problems.*

Legends

Tips, additional information etc

Important theorem, conclusion to remember.

Addition questions for further study.

My Comments and Notes

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The emphasis of this practice is to write the required equations or formulas **directly**.

You should remember all the conclusions in this practice.

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Coordinate Basics

Practice 1

(Distance) What is the distance between (x_1, y_1) and (x_2, y_2)?

Practice 2

(Middle Point) What is the coordinate of the middle point between (x_1, y_1) and (x_2, y_2)?

Practice 3

(Centroid) Give a triangle whose vertices are $(x_1, y_1), (x_2, y_2)$, and (x_3, y_3), respectively. What is the coordinate of its center of mass (centroid)?

Practice 4

(Incenter) Give a triangle whose vertices are $A(x_a, y_a), B(x_b, y_b)$, and $C(x_c, y_c)$, respectively. If $a = BC, b = CA$, and $c = BA$, what is the coordinate of its incenter?

Practice 5

(Interpolation) Give two points $A(x_a, y_a)$ and $B(x_b, y_b)$, a point C on $\overline{AB}$. If $AC : CB = m : n$ where m and n are two integers, find the coordinate of point C.

Practice 6

(Line by Two Points) If a straight line l passes two distinct points (x_1, y_1) and (x_2, y_2), what is l's equation?

Practice 7

(Line by Intercepts) What is the equation of a straight line if its x-intercept and y-intercept are a and b, respectively?

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Practice 8

(Plane by Intercepts) What is the equation of a plane if its x-intercept, y-intercept, and z-intercept are a, b, and c, respectively?

Practice 9

(Point to Line) What is the distance from the point (x_0, y_0) to the line $Ax + By + C = 0$?

Practice 10

(Point to Plane) What is the distance from the point (x_0, y_0, z_0) to the plane $Ax + By + Cz + D = 0$?

Practice 11

(Triangle Area) What is the area of a triangle whose vertices are $(x_1, y_1), (x_2, y_2)$, and (x_3, y_3)?

Practice 12

(Polygon Area) What is the area of a polygon whose vertices are $(x_1, y_1), (x_2, y_2), \cdots$ and (x_n, y_n)?

Practice 13

Given two parallel lines: $Ax + By + C_1 = 0$ and $Ax + By + C_2 = 0$, find the locus of all the points that are equidistant to these two lines.

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Coordinate Basics

Practice 14

(Circle Tangent Line) Let point $P(x_0, y_0)$ be on circle O, find the equation of the straight line which is tangent to O on point P when O is given by:

(i) $x^2 + y^2 = r^2$

(ii) $(x - a)^2 + (y - b)^2 = r^2$

(iii) $x^2 + y^2 + Dx + Ey + F = 0$

Practice 15

(Chord Passing Tangent Points) Let point $P(x_0, y_0)$ be outside the circle $O : x^2 + y^2 = 0$. If PA and PB are two lines that pass P and are tangent to O at A and B, find the equation of line AB.

Practice 16

(Distance to Tangent Points) Let point $P(x_0, y_0)$ be outside the circle O. If PA is a line that passes P and is tangent to O at A, find the distance between P and A when O is given by:

(i) $x^2 + y^2 = r^2$

(ii) $(x - a)^2 + (y - b)^2 = r^2$

(iii) $x^2 + y^2 + Dx + Ey + F = 0$

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Reference Solutions

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Coordinate Basics

Practice 1

(Distance) What is the distance between (x_1, y_1) and (x_2, y_2)?

$$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Practice 2

(Middle Point) What is the coordinate of the middle point between (x_1, y_1) and (x_2, y_2)?

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Practice 3

(Centroid) Give a triangle whose vertices are $(x_1, y_1), (x_2, y_2)$, and (x_3, y_3), respectively. What is the coordinate of its center of mass (centroid)?

$$\left(\frac{x_1 + x_2 + x_3}{x}, \frac{y_1 + y_2 + y_3}{3}\right)$$

Practice 4

(Incenter) Give a triangle whose vertices are $A(x_a, y_a), B(x_b, y_b)$, and $C(x_c, y_c)$, respectively. If $a = BC, b = CA$, and $c = BA$, what is the coordinate of its incenter?

$$\left(\frac{ax_a + bx_b + cx_c}{a + b + c}, \frac{ay_a + by_b + cy_c}{a + b + c}\right)$$

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Coordinate Basics

Practice 5

(Interpolation) Give two points $A(x_a, y_a)$ and $B(x_b, y_b)$, a point C on $\overline{AB}$. If $AC : CB = m : n$ where m and n are two integers, find the coordinate of point C.

$$\left(\frac{nx_a + mx_b}{m+n}, \frac{ny_a + my_b}{m+n}\right)$$

Practice 6

(Line by Two Points) If a straight line l passes two distinct points (x_1, y_1) and (x_2, y_2), what is l's equation?

$$\frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1} \quad or \quad \frac{y - y_2}{x - x_2} = \frac{y_2 - y_1}{x_2 - x_1}$$

Practice 7

(Line by Intercepts) What is the equation of a straight line if its x-intercept and y-intercept are a and b, respectively?

$$\frac{x}{a} + \frac{y}{b} = 1$$

Practice 8

(Plane by Intercepts) What is the equation of a plane if its x-intercept, y-intercept, and z-intercept are a, b, and c, respectively?

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$$

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Coordinate Basics

Practice 9

(Point to Line) What is the distance from the point (x_0, y_0) to the line $Ax + By + C = 0$?

$$\frac{\mid Ax_0 + By_0 + C \mid}{\sqrt{A^2 + B^2}}$$

Practice 10

(Point to Plane) What is the distance from the point (x_0, y_0, z_0) to the plane $Ax + By + Cz + D = 0$?

$$\frac{\mid Ax_0 + By_0 + Cz_0 + D \mid}{\sqrt{A^2 + B^2 + C^2}}$$

Practice 11

(Triangle Area) What is the area of a triangle whose vertices are $(x_1, y_1), (x_2, y_2)$, and (x_3, y_3)?

$$\frac{\mid (x_1y_2 - y_1x_2) + (x_2y_3 - y_2x_3) + (x_3y_1 - y_3x_1) \mid}{2}$$

Practice 12

(Polygon Area) What is the area of a polygon whose vertices are $(x_1, y_1), (x_2, y_2), \cdots$ and (x_n, y_n)?

$$\frac{\mid (x_1y_2 - y_1x_2) + (x_2y_3 - y_2x_3) + \cdots (x_ny_1 - y_nx_1) \mid}{2}$$

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Practice 13

Given two parallel lines: $Ax + By + C_1 = 0$ and $Ax + By + C_2 = 0$, find the locus of all the points that are equidistant to these two lines.

$$Ax + By + \frac{C_1 + C_2}{2} = 0$$

Practice 14

(Circle Tangent Line) Let point $P(x_0, y_0)$ be on circle O, find the equation of the straight line which is tangent to O on point P when O is given by:

(i) $x^2 + y^2 = r^2$

(ii) $(x - a)^2 + (y - b)^2 = r^2$

(iii) $x^2 + y^2 + Dx + Ey + F = 0$

(i) $x_0x + y_0y = r^2$

(ii) $(x_0 - a)(x - a) + (y_0 - b)(y - b) = r^2$

(iii) $x_0x + y_0y + D \cdot \frac{x_0 + x}{2} + E \cdot \frac{y_0 + y}{2} + F = 0$

Practice 15

(Chord Passing Tangent Points) Let point $P(x_0, y_0)$ be outside the circle $O : x^2 + y^2 = 0$. If PA and PB are two lines that pass P and are tangent to O at A and B, find the equation of line AB.

$$x_0x + y_0y = r^2$$

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Practice 16

(Distance to Tangent Points) Let point $P(x_0, y_0)$ be outside the circle O. If PA is a line that passes P and is tangent to O at A, find the distance between P and A when O is given by:

(i) $x^2 + y^2 = r^2$

(ii) $(x - a)^2 + (y - b)^2 = r^2$

(iii) $x^2 + y^2 + Dx + Ey + F = 0$

(i) $\sqrt{x_0^2 + y_0^2 - r^2}$

(ii) $\sqrt{(x_0 - a)^2 + (y_0 - b)^2 - r^2}$

(iii) $\sqrt{x_0^2 + y_0^2 + Dx_0 + Ey_0 + F}$

Tip: How can you remember these three formulas?

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Battle Field

Selective problems from recent comptitions:

Problem 1: 2016 AMC10B #20 (Ref 2925)

Problem 2: 2015 AIME II #9 (Ref 76)

Problem 3: 2015 AIME I #4 (Ref 56)

Problem 4: 2013 AMC12A #13 (Ref 489)

Problem 5: 2013 MathCounts State Target #6 (Ref 1841)

Problem 6: 2012 AMC10B #23 (Ref 1427)

Problem 7: 2012 MathCounts Chapter Target #3 (Ref 1975)

Problem 8: 2012 MathCounts Chapter Team #2 (Ref 1982)

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